

# Accretion, Growth of Supermassive Black Holes, and Feedback in Galaxy Mergers

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## ABSTRACT

Super-Eddington accretion is very efficient in growing the mass of a black hole: in a fraction of the Eddington time its mass can grow to an arbitrary large value if the feedback effect is not taken into account. However, since super-Eddington accretion has a very low radiation efficiency, people have argued against it as a major process for the growth of the black holes in quasars since observations have constrained the average accretion efficiency of the black holes in quasars to be  $\gtrsim 0.1$ . In this paper we show that the observational constraint does not need to be violated if the black holes in quasars have undergone a two-phase growing process: with a short super-Eddington accretion process they get their masses inflated by a very large factor until the feedback process becomes important, then with a prolonged sub-Eddington accretion process they have their masses increased by a factor  $\gtrsim 2$ . The overall average efficiency of this two-phase process is then  $\gtrsim 0.1$ , and the existence of black holes of  $10^9 M_\odot$  by redshift 6 is easily explained. Observational test of the existence of the super-Eddington accretion phase is briefly discussed.

## Key words:

black hole physics – accretion, accretion disks – galaxies: active – quasars: general – cosmology: miscellaneous.

## 1 INTRODUCTION

The very existence of black holes of masses  $10^9 M_\odot$  or more in quasars at cosmic redshift  $\gtrsim 6$  (Fan et al. 2001; Barth et al. 2003; Willott, McLure & Jarvis 2003) has greatly challenged the theory for the growth of supermassive black holes. Recently, a  $2 \times 10^9 M_\odot$  black hole has also been identified in a quasar at redshift 7 (Mortlock et al. 2011). The most natural way for the growth of black holes is accretion of gases. However, as discussed below, it is very unlikely that a black hole can grow to a mass of  $10^9 M_\odot$  by redshift 6 by accretion from a standard geometrically thin accretion disk.

An astronomical black hole has two independent parameters, its mass  $M_H$  and angular momentum  $J_H$ . A dimensionless spin parameter is defined by  $a_* \equiv cJ_H/GM_H^2$ , where  $G$  is the Newtonian gravitational constant, and  $c$  is the speed of light. According to general relativity,  $a_*$  must have an amplitude smaller than or equal to unity, i.e.,  $a_*^2 \leq 1$ .

A geometrically thin disk is very efficient in converting mass into radiation and spinning up a black hole. When the effect of photon recapture by the black hole is included, the maximum spin that a black hole can acquire by accretion from a thin accretion disk is  $a_* = 0.998$  (the “canonical”

spin, Thorne 1974), corresponding to a disk efficiency  $\approx 0.3$  (the efficiency in converting rest mass into radiated energy; see also Li et al. 2005). When a black hole is spun up from the non-rotating state ( $a_* = 0$ ) to the canonical state ( $a_* = 0.998$ ), its mass grows by a factor of  $\approx 2.7$ .

Thus, if a supermassive black hole had been growing by accretion from a thin disk, we would expect that its spin had already been equal to the canonical value from the very early stage, so that during the majority part of the history of the black hole growth the disk efficiency should be close to 0.3. With such a high disk efficiency, for a black hole to grow to a mass of  $10^9 M_\odot$  by the redshift  $z = 6$  through accretion from a thin disk, its initial mass should be  $\gtrsim 10^7 M_\odot$  (see Sec. 2). This is almost impossible since among the seed black holes that have been proposed, the less exotic ones are in the range of  $10^2 - 10^5 M_\odot$  (Volonteri & Rees 2005; Volonteri 2010; Alexander & Hickox 2011).

For a black hole to grow from a reasonable seed mass to a mass of  $10^9 M_\odot$  by  $z = 6$  through accretion, the disk efficiency has to be very low. Although accretion modes with low efficiency exist (e.g., accretion through a geometrically thick disk), observations have provided stringent constraint on the average accretion efficiency. Soltan (1982) has proposed a simple approach to constrain the disk accretion efficiency in quasars: dividing the observed integrated luminosity density of quasars by the observed local black

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hole mass density. Based on this approach, Yu & Tremaine (2002) found that the average disk accretion efficiency for black holes in quasars must be  $\gtrsim 0.1$ , otherwise the accreted mass density of the black holes during quasar phases would exceed the local mass density of black holes (see also Elvis, Risaliti & Zamorani 2002; Wang et al. 2006). This constraint indicates that the geometrically thin disk accretion phase must have occupied a significant fraction of time during the cosmic black hole accretion history.

Recently, disks misaligned with the black hole spin have been revisited by King et al. (2005). A major discovery is that when the disk angular momentum is less than twice of the black hole spin and the angle between them satisfies certain condition the alignment torque between the black hole and the disk will counter-align the disk to the black hole spin. Then, it is possible that under some favorable conditions accretion is composed of a series of episodes with the disk angular momentum randomly orientated relative to the black hole spin so that spinning-up and spinning-down of the black hole have about equal possibilities. For example, King, Pringle & Hofmann (2008) have proposed that if the disk is self-gravitating, it may repeatedly collapse producing accretion with a series of random orientations and thus the accretion will result in counter-alignment roughly half of time. Then, if the repeating process happens very frequently, the spin parameter of the black hole will fluctuate around a mean value near zero, indicating a lower efficiency in converting accretion mass into radiated energy than a maximally spinning black hole. For recent review and discussion on misaligned disks and their effects on evolution of the black hole spin, see Fanidakis et al. (2011).

However, Volonteri, Sikora & Lasota (2007) argued that within the cosmological framework, where the most massive black holes have grown in mass via merger-driven accretion, one expects that disk accretion tends to make most supermassive black holes in elliptical galaxies to have on average higher spins than black holes in spiral galaxies. They proposed that the evolution of supermassive black holes in elliptical galaxies are dominated by long and continuous accretion episodes arisen from major mergers so their spins are brought up to very high values. While in spiral galaxies, growth of black holes are probably dominated by random and small accretion episodes (e.g., tidally disrupted stars, accretion of molecular clouds) and hence the black holes on average have low value spins. They also argued that their models are in agreement with by the discovery that disk galaxies tend to be weaker radio sources than elliptical galaxies (Sikora, Stawarz & Lasota 2007).

Even though it is possible that the black hole may not get a large net spin through random accretion, it is still unlikely that accretion through geometrically thin disks is efficient enough to grow the mass of the black hole to  $> 10^9 M_\odot$  by  $z = 6$ . As will be shown in Sec. 2, for a geometrically thin disk around a non-spinning black hole with a luminosity 0.3 times the Eddington luminosity, to have the black hole mass  $3 \times 10^9 M_\odot$  by  $z = 6$  the initial mass of the seed black hole must be at least  $1.6 \times 10^5 M_\odot$ .

Growth of supermassive black holes by mergers has also been discussed in the literature (Kauffmann & Haehnelt 2000; Hughes & Blandford 2003; Shapiro 2005; O’Leary et al. 2006; Berti & Volonteri 2008). However, the black hole growth via merging is not expected to be

very effective, especially when the recoil speed caused by the emission of gravitational wave is large (O’Leary et al. 2006; Volonteri 2007).

Growth of black holes by super-Eddington accretion has been extensively explored (Kawaguchi 2003; Kawaguchi et al. 2004; Begelman, Volonteri & Rees 2006). In particular, with Monte Carlo simulation Volonteri & Rees (2005) have investigated a model for the early assembly of supermassive black holes by the merger tree of mini-halos, with a stable super-Eddington accretion phase at redshift  $\gtrsim 20$ . They found that even a short phase of super-Eddington accretion eases the requirement by the existence of black holes of masses  $\gtrsim 10^9 M_\odot$  in quasars at redshift  $\sim 6$ . Recent study on the growth of black holes by accretion and super-Eddington accretion includes Wang et al. (2008), Wyithe & Loeb (2011), and Park & Ricotti (2012).

Super-Eddington accretion, for which the mass accretion rate exceeds the Eddington limit but the luminosity does not, is very effective in feeding a black hole but has a very low radiation efficiency. Hence, people have argued against super-Eddington accretion by the observational constraint on the average accretion efficiency  $\gtrsim 0.1$  (Yu & Tremaine 2002; Elvis, Risaliti & Zamorani 2002; Wang et al. 2006).

In this paper, we perform an analytical investigation of the growth of supermassive black holes via joint sub- and super-Eddington accretion in the framework of coevolution of black holes, quasars, and galaxies. This framework, in which quasar activity is assumed to be triggered by mergers of galaxies (Di Matteo, Springel & Hernquist 2005; Hopkins et al. 2005, 2006, 2007; Malbon et al. 2007; Sijacki et al. 2007; Di Matteo et al. 2008; Treister et al. 2010; Debuhr, Quataert & Ma 2011), has been motivated by the observations that black hole masses in nearby galaxies correlate with some properties of the host galaxies, such as the bulge luminosities and masses (Kormendy & Richstone 1995; Magorrian et al. 1998), and the central velocity dispersion (Ferrarese & Merritt 2000; Gebhardt 2000). However, we point out that the arguments in this paper in favor of super-Eddington accretion are valid for any mechanism that drives gases rapidly toward the galactic nucleus and fuel the growth of the black hole. Galaxy mergers are only one of such mechanisms, other mechanisms include, for instance, global disk instabilities (Mo, Mao & White 1998; Cole et al. 2000; Bower et al. 2006).

We assume that super-Eddington accretion takes place during a major merger of galaxies, since a major merger is expected to bring a lot of cold gas to the nucleus of the merged galaxy. We show that, with super-Eddington accretion the mass of a black hole grows very rapidly. Within a fraction of  $10^8$  yr, the mass of the black hole can grow to an arbitrary large value, provided that there is enough gas to accrete. We show that if the feedback effect is taken into account, a super-Eddington accretion phase will switch to a sub-Eddington phase when the mass of the black hole becomes large enough ( $\gtrsim 10^8 M_\odot$ ), then a black hole of mass  $10^9 M_\odot$  can easily be produced by  $z = 6$ .

We argue that, since the super-Eddington phase can only take place in favorable conditions (e.g., during a major merger of galaxies), it can at most occupy a short period of time during the history of a supermassive black hole. Then, the accretion efficiency averaged over the whole accretion

history can easily reach a value  $\gtrsim 0.1$ , consistent with the observational constraint.

The paper is organized as follows. In Sec. 2 we write down the fundamental equations governing the evolution of the mass and the spin of a black hole under disk accretion, and show that the standard thin disk accretion is not efficient enough to grow a black hole. In Sec. 3 we estimate the mass accretion rate with the Bondi model, and show that accretion will remain super-Eddington if it is initially so. In Sec. 4, we review the solution for sub-Eddington accretion. In Sec. 5, we present the solution for super-Eddington accretion, and show that super-Eddington is very efficient for growing the black hole. In Sec. 6 we propose a two-phase scenario for the growth of supermassive black holes: with an initial super-Eddington phase and a later sub-Eddington phase, the mass of the black hole grows rapidly and the observational constraint on the average accretion efficiency is not violated. In Sec. 7, we present a simple model for the growth of supermassive black holes in quasars, with the feedback effect being taken into account. In Sec. 8 we draw our conclusions and briefly discuss the observational test of the existence of super-Eddington accretion phase in quasars and active galactic nuclei (AGNs).

We assume a flat  $\Lambda$ CDM cosmology with  $\Omega_m = 0.26$ ,  $\Omega_\Lambda = 0.74$ , and  $H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Dunkley et al. 2009).

## 2 GROWTH OF BLACK HOLES VIA ACCRETION: FUNDAMENTAL EQUATIONS

Let us express the mass of the black hole, the mass accretion rate, and time in dimensionless parameters

$$M_H = m M_{H,0}, \quad \dot{M} = \xi L_{\text{Edd}}/c^2, \quad t = \tau t_{\text{Edd}},$$

where  $M_{H,0}$  is the initial mass of the black hole,  $L_{\text{Edd}} = 1.3 \times 10^{38} (M_H/M_\odot) \text{ erg s}^{-1}$  is the Eddington luminosity, and

$$t_{\text{Edd}} \equiv \frac{M_H c^2}{L_{\text{Edd}}} = 4.51 \times 10^8 \text{ year}$$

is the Eddington time.

Note, our definition of the Eddington time,  $t_{\text{Edd}}$ , differs from the Salpeter time by a factor  $\varepsilon$ , the efficiency of accretion. When  $\varepsilon = 0.1$ ,  $t_{\text{Edd}}$  is ten times of the Salpeter time. Although the Salpeter time is more often used in the literature, we find that  $t_{\text{Edd}}$  is more convenient, at least for the purpose of the current paper.

Then, the evolution of the mass and the spin of the black hole under disk accretion is determined by (Thorne 1974)

$$\frac{dm}{d\tau} = \xi m E_{\text{in}}^\dagger, \quad (1)$$

$$\frac{da_*}{d\tau} = \xi \left( \frac{L_{\text{in}}^\dagger}{M_H} - 2a_* E_{\text{in}}^\dagger \right), \quad (2)$$

where  $E_{\text{in}}^\dagger$  is the specific energy,  $L_{\text{in}}^\dagger$  is the specific angular momentum of a disk particle at the inner boundary of the disk.

The efficiency of the disk in converting rest mass into

energy is  $\varepsilon = 1 - E_{\text{in}}^\dagger$ . The luminosity of the disk is

$$L = \varepsilon \dot{M} c^2 = \varepsilon \xi L_{\text{Edd}}. \quad (3)$$

In the theory of accretion disks it is usually assumed that the luminosity of the disk is bounded by the Eddington luminosity. The arguments are that if the luminosity of the disk radiation exceeds the Eddington luminosity, it is expected that the intense pressure of the disk radiation will halt the accretion flow so that steady accretion with luminosity greater than the Eddington luminosity is impossible (Frank, King & Raine 2002). So, we assume that the luminosity is bounded by the Eddington luminosity, i.e.<sup>1</sup>

$$\varepsilon \xi \leq 1. \quad (4)$$

If the disk is geometrically thin, its particles move on Keplerian orbits in the equatorial plane.<sup>2</sup> Then, the inner boundary of the disk is at the marginally stable circular orbit, i.e.,  $r_{\text{in}} = r_{\text{ms}}$  (Page & Thorne 1974). The disk efficiency is then  $\varepsilon_0 = \varepsilon(r_{\text{ms}}) = 1 - E_{\text{ms}}^\dagger$ . This efficiency is a function of the spin of the black hole. When  $a_* = 0$ , the efficiency is  $\varepsilon_0 \approx 0.06$ . When  $a_* = 0.998$ , the maximum spin that a black hole can get through thin disk accretion, the efficiency is  $\varepsilon_0 \approx 0.3$ .

Substituting  $\varepsilon = \varepsilon_0$  into the inequality (4), we get a limit on the mass accretion rate for thin disk accretion:  $\xi \lesssim \xi_{\text{cr}}$ , where

$$\xi_{\text{cr}} \equiv \frac{1}{\varepsilon_0}. \quad (5)$$

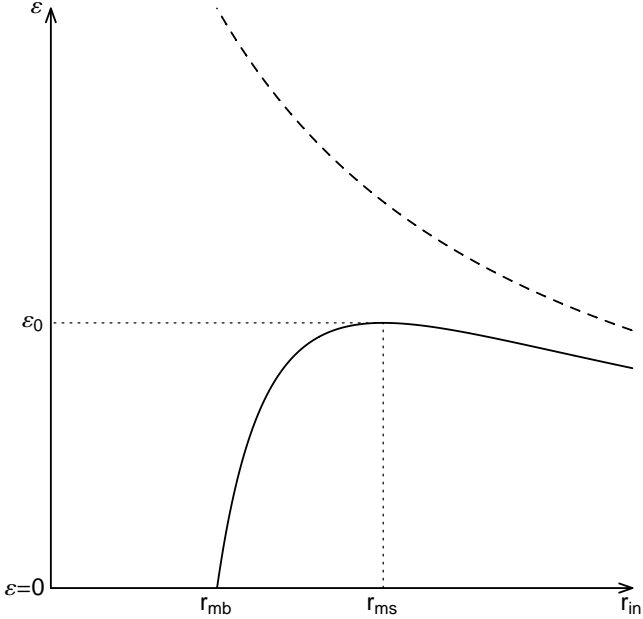
Hence, a necessary condition for the disk to be geometrically thin is that  $\xi < \xi_{\text{cr}}$  (i.e., the accretion rate must be sub-Eddington) since then the disk luminosity is below the Eddington limit.<sup>3</sup>

When  $\xi > \xi_{\text{cr}}$ , the disk has a super-Eddington accretion rate, and the inner region of the disk must be geometrically thick due to the trap of radiation (Begelman 1979; Abramowicz & Lasota 1980). In this case, the inner boundary of the disk is located at a place between the marginally stable orbit and the marginally bound orbit (i.e.,  $r_{\text{mb}} < r_{\text{in}} < r_{\text{ms}}$ ), and its efficiency is smaller than  $\varepsilon_0$  so that the luminosity does not exceed the Eddington limit (Abramowicz & Lasota 1980; Paczyński 1982; Paczyński & Abramowicz 1982).

<sup>1</sup> Detailed calculations on supercritical accretion indicate that when the mass accretion rate exceeds the critical mass accretion rate defined by the Eddington luminosity (i.e., when  $\xi > \xi_{\text{cr}}$ ,  $\xi_{\text{cr}}$  is defined by eq. 5), the luminosity of the disk is not cut-off abruptly by the Eddington luminosity. Instead, above the critical mass accretion rate the disk luminosity varies logarithmically with the mass accretion rate and about  $10L_{\text{Edd}}$  can be approached for very high mass accretion rate (Abramowicz et al. 1988; Watarai 2006).

<sup>2</sup> Even if initially the disk is not in the equatorial plane, the Bardeen-Petterson effect will cause the inner region of the disk to have its angular momentum to be aligned (Bardeen & Petterson 1975) or counter-aligned (King et al. 2005) to the spin of the black hole quickly.

<sup>3</sup> Here we do not consider the exotic model of geometrically thick disks with very low luminosity, like the ADAF model of Narayan & Yi (1994). ADAF is not expected to operate in the early stage of galaxy evolution (R. Narayan, private communications).



**Figure 1.** The efficiency of a relativistic disk around a Kerr black hole as a function of the radius of the disk inner boundary (solid line). The efficiency peaks at  $r_{ms}$ , the marginally stable orbit, and drops to zero at  $r_{mb}$ , the marginally bound orbit. For comparison, the efficiency of a Newtonian disk is shown with a dashed curve, which monotonically increases as the radius of the disk inner boundary decreases.

Even for a geometrically thick accretion disk, the specific energy and the specific angular momentum of disk particles are still approximately equal to the Keplerian values at the inner boundary of the disk since where the gas and radiation pressure is always negligible, although this is not the case for particles on orbits of larger radii. Thus, we always set  $E_{in}^\dagger$  and  $L_{in}^\dagger$  to their Keplerian values as though particles on the inner boundary move on a Keplerian circular orbit.

The specific energy  $E^\dagger$  for particles on Keplerian circular orbit in the equatorial plane of a Kerr black hole minimizes at the marginally stable orbit  $r = r_{ms}$ , and  $E^\dagger = 1$  at the marginally bound orbit  $r = r_{mb}$  (Bardeen, Press & Teukolsky 1972). Hence,  $\varepsilon$  maximizes when the disk inner boundary is at the marginally stable orbit, and  $\varepsilon = 0$  at the marginally bound orbit (Fig. 1). For an efficiency between zero and the maximum value  $\varepsilon_0$ , the inner boundary of the disk is at a radius between  $r_{mb}$  and  $r_{ms}$ . For comparison, in Fig. 1 we also show the efficiency of a Newtonian disk, which monotonically increases as the radius of the disk inner boundary decreases.

For super-Eddington accretion ( $\xi > \xi_{cr}$ ), the luminosity is  $L = L_{Edd}$ . Thus, the disk efficiency is  $\varepsilon = 1/\xi$ . In the case of extreme super-Eddington accretion with  $\xi \gg 1$ , the efficiency  $\varepsilon \ll 1$  so we must have  $r_{in} \approx r_{mb}$ .

In study of the growth of supermassive black holes, people often assume that the disk is geometrically thin (so its inner boundary is at the marginally stable orbit), and the ratio of the disk luminosity to the Eddington luminosity is a constant number smaller than but close to unity. That is,  $\mu \equiv \varepsilon_0 \xi = \text{constant}$ , and  $0.1 \lesssim \mu \lesssim 1$ . In this case, equa-

tion (1) becomes

$$\frac{d \ln m}{d\tau} = \frac{\mu(1 - \varepsilon_0)}{\varepsilon_0}. \quad (6)$$

When  $\varepsilon_0 = \text{constant}$ , the solution to equation (6) is

$$m(\tau) = \exp \left[ \frac{\mu(1 - \varepsilon_0)}{\varepsilon_0} \tau \right], \quad (7)$$

where we have set  $m = 1$  at  $\tau = 0$ . Under thin disk accretion the spin of the black hole will quickly grow to the canonical value  $a_* = 0.998$  and then is saturated at that value, when the effect of photon recapture is considered (Thorne 1974; Li et al. 2005). For a black hole to spin up from a non-rotating state to the canonical state, the mass of the black hole need only increase by a factor of about 2.7. Hence, the most natural value for a constant  $\varepsilon_0$  is the efficiency when the black hole is in the canonical state, which is  $\varepsilon_0 = \varepsilon_{\max} \approx 0.3$ . Then, the solution becomes  $m(\tau) \approx e^{2.3\mu\tau}$ .

At redshift  $z = 6$ , the age of the universe is about 0.96 Gyr, and  $\tau \approx 2.1$ . Thus, since  $\mu \lesssim 1$ , at  $z = 6$  we should have  $m \lesssim 130$ . To explain the existence of a black hole of mass  $3 \times 10^9 M_\odot$  at  $z = 6$  (Willott, McLure & Jarvis 2003) by thin disk accretion, the mass of the seed black hole would have to be  $\gtrsim 2 \times 10^7 M_\odot$ . This is almost impossible since among the seed black holes that have been proposed the less exotic ones are in the range of  $10^2 - 10^5 \odot$  (Volonteri & Rees 2005; Volonteri 2010; Alexander & Hickox 2011).

For a geometrically thin disk to be a good approximation, the disk luminosity should not exceed 0.3 times the Eddington luminosity (McClintock et al. 2006). Take  $\mu = 0.3$  and  $\varepsilon_0 = 0.06$  (the efficiency for a non-spinning black hole), we get  $m(\tau) \approx e^{4.7\tau}$ . Then, to explain the existence of a black hole of mass  $3 \times 10^9 M_\odot$  at  $z = 6$  by the growth of black holes via the model of a thin disk around an always-non-spinning black hole, the mass of the seed black hole would have to be  $\sim 1.6 \times 10^5 M_\odot$ . This would also require a quite exotic high-mass seed black hole.

### 3 THE MASS ACCRETION RATE

The assumption  $\mu = \varepsilon_0 \xi = \text{constant}$  in the standard treatment may not be correct in reality. In fact, the mass accretion rate is determined by the boundary condition of the accretion flow. A widely adopted formula for estimation of the mass accretion rate is given by the Bondi rate (Bondi & Hoyle 1944; Bondi 1952; Novikov & Thorne 1973; Shapiro & Teukolsky 1983)

$$\dot{M} = 4\pi\lambda_s\rho_\infty \frac{G^2 M_H^2}{c_{s,\infty}^3},$$

where  $\rho_\infty$  and  $c_{s,\infty}$  are, respectively, the mass density and the sound speed at infinity, and  $\lambda_s \sim 1$  is a number depending on the state equation of the gas.

The Bondi rate was derived from a steady and spherical accretion flow. For the case of disk accretion, the accretion flow cannot be spherically symmetric and steady even at a distance far from the central black hole. Hopkins & Quataert (2010) have shown that the disk accretion flow is highly nonsymmetric and the accretion rate is highly time variable for a given set of conditions in the galaxy at  $\sim \text{kpc}$ . Recently, Hobbs et al. (2012) have shown

that when the gas is in a state of free-fall at the evaluation radius due to efficient cooling and the dominant gravity of the surrounding halo, the Bondi model cannot give correct estimation of the accretion rate. They have proposed an expression for the sub-grid accretion rate which interpolates between the free-fall regime and the Bondi regime by taking into account the contribution of the halo to the gas dynamics.

If  $\rho_\infty$  and  $c_{s,\infty}$  are constants in time, by the Bondi formula we have  $\dot{M} \propto M_H^2$ . Since  $L_{\text{Edd}} \propto M_H$ , by the definition of  $\xi$  we have  $\xi \propto M_H \propto m$ . The formula proposed by Hobbs et al. (2012) takes into account the gravity of the halo, which results a mass accretion rate that varies with  $M_H$  slower than that given by the Bondi rate. Therefore, to make the results more general, we assume that

$$\xi = \xi_0 m^\gamma, \quad (8)$$

where  $0 \leq \gamma \leq 1$ , and  $\xi_0$  is the value of  $\xi$  at the beginning of accretion ( $m = 1$ ). The limit of  $\gamma = 1$  corresponds to the Bondi model. The limit of  $\gamma = 0$  corresponds to the case that the black hole accretes at a rate given by the Eddington rate multiplied by a constant number, which corresponds to a constant ratio of the disk luminosity to the Eddington luminosity when the disk efficiency  $\varepsilon$  remains a constant. A value of  $\gamma$  between 0 and 1 may represent a more realistic case, for instance the model of Hobbs et al. (2012).

Suppose the black hole is always spun up by accretion, i.e.,  $a_* > 0$  does not decrease with time. Then,  $\varepsilon_0$  does not decrease with time. If at the beginning the accretion is super-Eddington (i.e.,  $\xi_0 > \xi_{\text{cr}}$ ), it will remain super-Eddington as accretion goes on as long as  $\gamma \geq 0$ , until some feedback effect becomes important to reduce the accretion rate, or the fuel is finally exhausted. Even if at the beginning the accretion is sub-Eddington (i.e.,  $\xi_0 < \xi_{\text{cr}}$ ) it will eventually becomes super-Eddington if  $\gamma > 0$  provided that the accretion takes place for a sufficiently long time.

It is under debate how much a fraction of the total inflowing gas is expelled into an outflow when the total mass accretion rate is super-Eddington (Shakura & Sunyaev 1973; Ogilvie & Livio 1998, 2001; Blandford & Begelman 1999; Lipunova 1999). The concept of Eddington luminosity was derived by the consideration that when the luminosity exceeds the Eddington limit the intense pressure of the radiation on the ionized gas would overcome the gravity of the central object onto which the gas accreted so that the accretion would be halted until the luminosity got below the Eddington limit. However, since  $L = \varepsilon \dot{M} c^2$ , the reduce in the luminosity does not necessarily requires the reduce in the mass accretion rate, it can also be realized by reducing the accretion efficiency  $\varepsilon$ .

Even for spherical accretion, it was found that an intense outflow may not be produced when the accretion rate exceeds the Eddington limit (Begelman 1979). As the accreting gas falls deep enough in the gravitational well of the central black hole so that the Eddington luminosity is approached, a “trapping radius” of radiation is crossed within which the radiation is trapped in the gas and dragged into the black hole by the inflowing gas. Effectively, the accretion efficiency is reduced while the mass accretion rate is not. (For recent discussion on photon trapping, see Wyithe & Loeb 2011.)

The situation is similar for the case of accre-

tion disks (Abramowicz & Lasota 1980; Abramowicz et al. 1988). When the mass accretion rate exceeds the Eddington limit, a “trapping radius” is also formed in the inner region of the disk. Within the trapping radius the disk radiation is trapped in the gas and dragged into the central black hole by the accreting gas, the effective accretion efficiency is hence reduced. In addition to this, general relativity also provides a new possibility for reducing the disk efficiency. Because of general relativity, the gravitational binding energy decreases as the inner boundary of the disk moves inward inside the marginally stable orbit, so that the disk efficiency is decreased (Fig. 1). As the inner boundary approaches the marginally bound orbit, the disk efficiency approaches zero. Thus, to prevent the growth of the disk luminosity, it is not necessary to reduce the mass accretion rate since it can be realized by the photon trapping effect and/or the general relativistic effect.

The ability of preventing the disk luminosity from exceeding the Eddington luminosity by lowering the radiative efficiency without need of reducing the mass rate accreting onto the central black hole leads to the possibility that in the black hole case a highly super-Eddington accretion may not necessarily lead to a large outflow of mass. Therefore, in this paper we ignore the effect of outflows for super-Eddington accretion, in which case the inner boundary of the disk will move inward toward the marginally bound orbit to reduce the disk efficiency so that the luminosity does not exceeds the Eddington luminosity.

Substituting equation (8) into equations (1) and (2), we obtain

$$\frac{dm}{d\tau} = \xi_0 m^{\gamma+1} E_{\text{in}}^\dagger, \quad (9)$$

$$\frac{da_*}{d\tau} = \xi_0 m^\gamma \left( \frac{L_{\text{in}}^\dagger}{M_H} - 2a_* E_{\text{in}}^\dagger \right). \quad (10)$$

#### 4 SUB-EDDINGTON ACCRETION

Under sub-Eddington accretion, the disk is geometrically thin and the inner boundary is fixed at the marginally stable orbit:  $r_{\text{in}} = r_{\text{ms}}$ . Then,  $E_{\text{in}}^\dagger = E_{\text{ms}}^\dagger$  and  $L_{\text{in}}^\dagger/M_H = L_{\text{ms}}^\dagger/M_H$  are functions of  $a_*$  only, equations (9) and (10) can be solved for  $m(\tau)$  and  $a_*(\tau)$ .

The ratio of equation (9) and equation (10) leads to a solution for  $m(a_*)$

$$m(a_*) = \exp \left[ \int_{a_{*,0}}^{a_*} \frac{E_{\text{ms}}^\dagger da_*}{L_{\text{ms}}^\dagger/M_H - 2a_* E_{\text{ms}}^\dagger} \right],$$

which is independent of  $\gamma$ . With the solution of  $m(a_*)$ , we can solve for  $\tau$  from (10)

$$\tau(a_*) = \frac{1}{\xi_0} \int_{a_{*,0}}^{a_*} \frac{m(a_*)^{-\gamma} da_*}{L_{\text{ms}}^\dagger/M_H - 2a_* E_{\text{ms}}^\dagger}.$$

The solution of  $m(a_*)$  leads to that when a black hole is spun up from  $a_* = 0$  to  $a_* = 0.998$ , its mass is increased by a factor of 2.2. This is somewhat smaller than the value when the effect of photon recapture is considered, the latter is about 2.7 (Thorne 1974).

The solution of  $\tau(a_*)$  shows that, for a black hole to spin up from  $a_* = 0$  to  $a_* = 0.998$ , it needs a time  $\Delta t \approx 0.935 \xi_0^{-1} t_{\text{Edd}}$  when  $\gamma = 0$ , or  $\Delta t \approx 0.635 \xi_0^{-1} t_{\text{Edd}}$  when  $\gamma =$

1. If instead we assume that  $\mu = \varepsilon_0 \xi = \text{constant}$  during the accretion, we will have  $\Delta t \approx 0.146 \mu^{-1} t_{\text{Edd}}$ . If the effect of photon recapture is considered, the value of  $\Delta t$  should be somewhat larger.

Therefore, by accretion the black hole is spun up to a limit spin  $a_{*,\text{lim}} = 0.998$  in a finite time, accreting a finite amount of mass. Afterwards, if the accretion rate remains sub-Eddington and the disk remains geometrically thin,  $a_*$  will stay at  $a_{*,\text{lim}}$ , and  $E_{\text{in}}^\dagger = E_{\text{lim}}^\dagger \equiv E_{\text{ms}}^\dagger(a_{*,\text{lim}}) \approx 0.7$ . Then, the solution to equation (9) is

$$m(\tau) = \frac{1}{(1 - \gamma \xi_0 E_{\text{lim}}^\dagger \tau)^{1/\gamma}}, \quad (11)$$

where we have set  $m = 1$  at  $\tau = 0$ . Note, when  $\gamma > 0$ ,  $m$  becomes infinite at  $\tau = 1/\gamma \xi_0 E_{\text{lim}}^\dagger$ .

However, the solution in equation (11) is valid only when  $\xi < \xi_{\text{cr}}$ , i.e., when the accretion is sub-Eddington. Since  $\xi(\tau) = \xi_0 m(\tau)^\gamma$ , the accretion becomes super-Eddington (i.e.,  $\xi > \xi_{\text{cr}}$ ) at  $\tau = \tau_s$ , where

$$\tau_s = \frac{1 - \xi_0 \varepsilon_{\text{lim}}}{\gamma \xi_0 E_{\text{lim}}^\dagger},$$

here  $\varepsilon_{\text{lim}} \equiv \varepsilon_0(a_{*,\text{lim}}) \approx 0.3$ .

Hence, the solution in equation (11) holds only when  $0 < \tau < \tau_s$ . By the time when the accretion becomes super-Eddington, the mass of the black hole has increased by a factor of  $m = (\xi_{\text{cr}}/\xi_0)^{1/\gamma}$ .

If at  $\tau = 0$  we have  $\xi = \xi_0 \ll 1$ , then  $\tau_s \approx 1/\gamma \xi_0 E_{\text{lim}}^\dagger \approx 1.4/\gamma \xi_0 \gg 1$ , it would need a time much longer than the Eddington time to reach the super-Eddington phase. Hence, sub-Eddington accretion is very inefficient to grow the black hole.

As  $\gamma \rightarrow 0$ , the solution in equation (11) becomes that in equation (7).

## 5 SUPER-EDDINGTON ACCRETION

Under super-Eddington accretion, for which  $\xi > \xi_{\text{cr}}$ , the inner boundary of the disk is at a radius between  $r_{\text{mb}}$  and  $r_{\text{ms}}$ . If the disk luminosity is limited by the Eddington luminosity, then  $\varepsilon \xi = 1$  (see the discussion in Sec. 2), from which we have

$$E_{\text{in}}^\dagger = 1 - \frac{1}{\xi}. \quad (12)$$

Equation (12) determines the radius at the inner boundary of the disk, for any given value of  $\xi$ .

Substituting equation (12) into equation (9), we obtain

$$\frac{dm}{d\tau} = \xi_0 m^{\gamma+1} - m. \quad (13)$$

The integration of equation (13) leads to

$$m(\tau) = \frac{1}{[\xi_0 - (\xi_0 - 1)e^{\gamma\tau}]^{1/\gamma}} \quad (14)$$

for  $0 < \tau < \tau_\infty$ , where

$$\tau_\infty \equiv \frac{1}{\gamma} \ln \frac{\xi_0}{\xi_0 - 1}. \quad (15)$$

The mass  $m$  becomes infinite at  $\tau = \tau_\infty$ . Thus, if  $\gamma > 0$ , the black hole would have accreted an infinite amount of mass in a finite time.

Note, the solution in equation (14), as well as the value of  $\tau_\infty$ , does not explicitly depend on the initial spin of the black hole. The solution depends only on  $\xi_0$  and  $\gamma$ .

For the accretion to be always super-Eddington, at  $\tau = 0$  we must have  $\xi_0 > 1/\varepsilon_0$ . Since  $\varepsilon_0 < \varepsilon_0(a_* = 1) \approx 0.42$ , we must have  $\xi_0 > 2.4$ . Thus,  $\tau_\infty$  always satisfies the constraint

$$\tau_\infty < \frac{1}{\gamma} \ln \frac{1}{1 - \varepsilon_0(a_* = 1)} \approx \frac{0.55}{\gamma}.$$

In the limit  $\xi \gg 1$ , by equation (12) we have  $E_{\text{in}}^\dagger \approx 1$ , so  $r_{\text{in}} \approx r_{\text{mb}}$  and  $L_{\text{in}}^\dagger \approx 2(r_{\text{mb}}/M_{\text{H}})^{1/2}$  (Bardeen, Press & Teukolsky 1972; Abramowicz & Lasota 1980). Equation (10) then becomes

$$\frac{da_*}{d\tau} = 2\xi_0 m^\gamma \left[ \left( \frac{r_{\text{mb}}}{M_{\text{H}}} \right)^{1/2} - a_* \right]. \quad (16)$$

The equation for the mass is simplified to

$$\frac{dm}{d\tau} = \xi_0 m^{\gamma+1}. \quad (17)$$

The solution to equation (17) is

$$m(\tau) = \frac{1}{(1 - \gamma \xi_0 \tau)^{1/\gamma}}, \quad (18)$$

which is consistent with the limit of equation (14) since then  $\tau_\infty \approx 1/\gamma \xi_0 \ll 1$ .

The ratio of equation (17) and equation (16) leads to

$$\int_1^m \frac{dm}{m} = \frac{1}{2} \int_{a_{*,0}}^{a_*} \frac{da_*}{(r_{\text{mb}}/M_{\text{H}})^{1/2} - a_*}, \quad (19)$$

whose solution is

$$m(a_*) = \exp \left[ \frac{1}{2} \int_{a_{*,0}}^{a_*} \frac{da_*}{(r_{\text{mb}}/M_{\text{H}})^{1/2} - a_*} \right], \quad (20)$$

independent of  $\gamma$ .

With super-Eddington accretion, the effect of photon recapture on the evolution of the black hole is not important since the radiation efficiency of the disk is very low. Then, the black hole can be spun up to a state with a spin arbitrarily close to unity (Abramowicz & Lasota 1980).

When  $a_{*,0} = 0$  and  $a_* = 1$ , by equation (20) and the equation (2.19) of Bardeen, Press & Teukolsky (1972) for  $r_{\text{mb}}$ , we have  $m(1) = 2$ . That is, when the black hole is spun up from  $a_* = 0$  to  $a_* = 1$  through super-Eddington accretion, its mass is doubled. In the limit of  $\xi \gg 1$  we have  $\tau_\infty \approx 1/\gamma \xi_0$ , then by equation (18) the time needed for a black hole to spin up from  $a_* = 0$  to  $a_* = 1$  is  $\approx (1 - 2^{-\gamma})\tau_\infty$ .

We note that, although super-Eddington accretion in principle can spin up the spin of the black hole all the way to a value arbitrarily close to unity, the subsequent sub-Eddington accretion will lower the spin of the black hole to the canonical value 0.998 by the effect of photon recapture.

## 6 A TWO-PHASE SCENARIO FOR THE GROWTH OF BLACK HOLES: THE FEEDBACK EFFECT

A main result in the last section is that the mass of a black hole can grow very rapidly via super-Eddington accretion. Within a fraction of the Eddington time, the black hole would have taken all the mass of the surrounding gas.

However, the growth of the black hole is limited by the feedback process (Silk & Rees 1998; Haehnelt, Natarajan & Rees 1998; Fabian 1999; King 2003, 2005). When the mass of the black hole is sufficiently large, the dynamical effect of the disk radiation and/or outflow on the surrounding gas will become important. The ambient gas will be swept away by the intense radiation and/or outflow from the central black hole, resulting that the mass accretion rate drops quickly. The accretion then transits from the super-Eddington phase to the sub-Eddington phase, the growth of the black hole is slowed down, and finally stops when the fuel is exhausted.

The feedback process produced by the intense disk radiation becomes important when (Silk & Rees 1998)

$$M_H \gtrsim 2 \times 10^8 M_\odot \left( \frac{f}{0.05} \right)^{-1} \left( \frac{\sigma}{400 \text{ km s}^{-1}} \right)^5,$$

where  $\sigma$  is the velocity dispersion of the halo hosting the galaxy,  $f$  denotes the fraction of the black hole radiation converted to the kinetic energy of the gas. While in models where the outflowing gas can efficiently cool such that the flow is dominated by momentum, a different relationship between  $M_H$  and  $\sigma$  was obtained (King 2003, 2005)

$$M_H \sim 10^8 M_\odot \left( \frac{\sigma}{200 \text{ km s}^{-1}} \right)^4.$$

This relation is remarkably close to the observed  $M_H - \sigma$  relation (Tremaine et al. 2002). No matter which feedback model is more precise, the important point is that when the mass of the black hole becomes greater than a few  $10^8 M_\odot$  the feedback process comes in to halt accretion.

Although super-Eddington accretion is very efficient in growing the black hole mass, it has a very low efficiency in converting mass into radiation. The instant efficiency of super-Eddington accretion is simply  $\varepsilon = 1/\xi$ , which is  $\ll 1$  when  $\xi \gg 1$ . The average efficiency for a process of super-Eddington accretion is given by (see eq. 21 below)

$$\bar{\varepsilon} = \frac{\int m d\tau}{\int \xi m d\tau}.$$

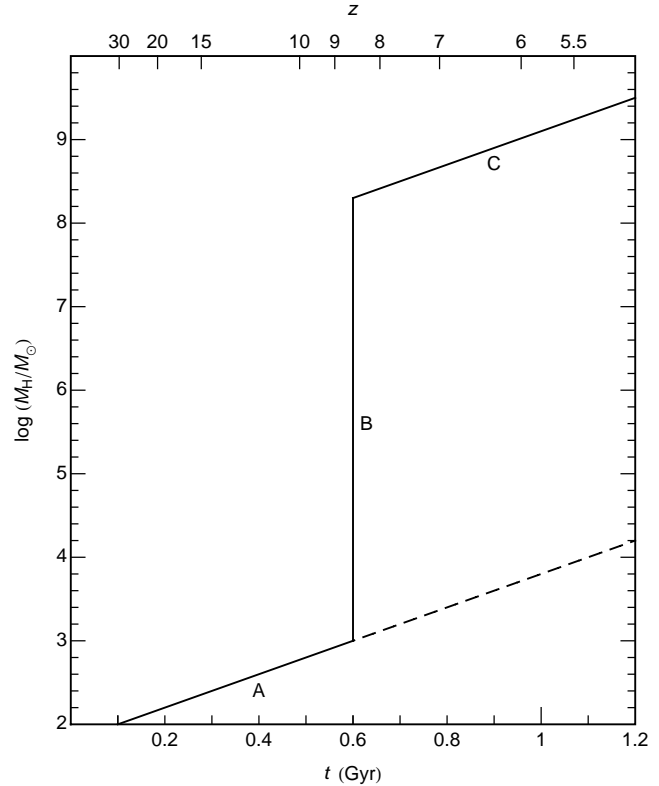
By equation (13), in the limit  $\xi \gg 1$  we get

$$\bar{\varepsilon} \approx \frac{1}{\xi} \times \begin{cases} \ln m, & (\gamma = 1); \\ \frac{1}{1-\gamma}, & (0 < \gamma < 1); \end{cases}$$

which is  $\ll 1$ .

However, this does not necessarily mean that the observational constraint  $\bar{\varepsilon} \gtrsim 0.1$  is violated. This is because of the following fact: super-Eddington accretion can only happen under very favorable conditions, in an environment with a lot of cold gas around the central black hole, which could happen, e.g., during a major merger of galaxies. Hence, accretion in super-Eddington phase can at most occupy a short transient period (up to a fraction of the Eddington time) in the whole evolution history of a supermassive black hole. The rest of the history would be dominated by sub-Eddington accretion, which has a high efficiency in converting mass into energy.

Therefore, we propose the following two-phase scenario for the growth of a supermassive black hole. Imagine that at an early time of the universe, e.g., at a redshift  $z = 20 - 30$ , a seed black hole of  $100 M_\odot$  was produced via collapse of a massive first generation star. The black hole grew slowly by



**Figure 2.** A sketch for the possible accretion history of a supermassive black hole (solid curve). At the cosmic time  $t = 0.1$  Gyr, a black hole of  $100 M_\odot$  starts sub-Eddington accretion (A;  $\mu = \varepsilon_0 \xi = \text{constant}$ ). At  $t = 0.6$  Gyr, the black hole enters a phase of super-Eddington accretion (B; triggered by, e.g., a major merger of galaxies), within a short period of time ( $\ll 1$  Gyr) its mass grows to  $10^{8.3} M_\odot$ . Then, the super-Eddington accretion ends (caused by the feedback effect, presumably), and the accretion becomes sub-Eddington again (C). By  $z = 6$  ( $t = 0.96$  Gyr) the black hole has a mass of  $10^9 M_\odot$ . The dashed curve denotes the extension of the initial sub-Eddington accretion, by which the mass of the black hole grows slowly. The horizontal axis is linear in time. The vertical axis is logarithm in the mass of the black hole. For references, the cosmic redshift from  $z = 5.5$  to  $z = 30$  is labeled on the top of the figure.

accreting mass from the surrounding gas. After a long period of this primary sub-Eddington accretion stage, the mass of the black increased to  $10^3 - 10^4 M_\odot$  before the surrounding environment of the black hole was changed suddenly caused by a major merger of galaxies. The merger supplied a large amount of cold gas to the neighbor of the central black hole, and the black hole entered a phase of super-Eddington accretion. The mass of the black hole then grew very rapidly, until it was large enough that the feedback process took place to decrease the mass accretion rate and slow down the growth of the black hole. The black hole then entered again a phase of sub-Eddington accretion.

The super-Eddington phase took a very short period of time (e.g.,  $0.05 t_{\text{Edd}}$  if at the beginning of the super-Eddington accretion we have  $\xi = 20$ ), but during which the mass of the black hole could inflate by a factor  $\gg 1$ . After the super-Eddington phase ended, the black hole continued growing slowly by sub-Eddington accretion. After a somewhat long time (e.g., several to ten Eddington time), the

mass of the black hole increased by a factor  $\zeta$ , which is of order 1 – 10. Although this second phase of sub-Eddington accretion is not very efficient for growing the mass of the black hole, it is crucial for boosting the average efficiency in converting mass into radiation.

For the whole accretion process, the average efficiency in converting mass into energy is calculated by

$$\bar{\varepsilon}(t) = \frac{\int_{t_0}^t L dt}{c^2 \int_{t_0}^t \dot{M} dt} = \frac{\int_{\tau_0}^{\tau} \varepsilon \xi m d\tau}{\int_{\tau_0}^{\tau} \xi m d\tau}, \quad (21)$$

where  $L = \varepsilon \dot{M} c^2$  is the luminosity of the disk.

For the super-Eddington accretion phase which boosts the mass of the black hole from  $m = 1$  to  $m = m_1 \gg 1$ , by equation (17) it can be calculated that

$$\int \xi m d\tau = \xi_0 \int m^{\gamma+1} d\tau = \int dm = m_1 - 1 \approx m_1,$$

and by  $\varepsilon \xi = 1$

$$\begin{aligned} \int \varepsilon \xi m d\tau &= \int m d\tau = \frac{1}{\xi_0} \int m^{-\gamma} dm \\ &= \frac{1}{(1-\gamma)\xi_0} (m_1^{1-\gamma} - 1) \approx \frac{m_1}{(1-\gamma)\xi_1}, \end{aligned}$$

where  $\xi_1 \equiv \xi_0 m_1^\gamma \gg 1$ .

For the sub-Eddington process which boosts the mass of the black hole from  $m = m_1$  to  $m = \zeta m_1$  with a constant efficiency  $\varepsilon = \varepsilon_0 = 1 - E_{\text{in}}^\dagger$ , by equation (1) we have

$$\int \xi m d\tau = \frac{1}{E_{\text{in}}^\dagger} \int dm = \frac{(\zeta - 1)m_1}{1 - \varepsilon_0},$$

and

$$\int \varepsilon \xi m d\tau = \varepsilon_0 \int \xi m d\tau = \frac{\varepsilon_0(\zeta - 1)m_1}{1 - \varepsilon_0}.$$

Hence, for the entire accretion process we have

$$\int \xi m d\tau \approx m_1 + \frac{(\zeta - 1)m_1}{1 - \varepsilon_0} = \frac{\zeta - \varepsilon_0}{1 - \varepsilon_0} m_1,$$

and

$$\int \varepsilon \xi m d\tau \approx \frac{m_1}{(1-\gamma)\xi_1} + \frac{\varepsilon_0(\zeta - 1)m_1}{1 - \varepsilon_0} \approx \frac{\varepsilon_0(\zeta - 1)}{1 - \varepsilon_0} m_1,$$

since  $\xi_1 \gg 1$ . Therefore the average efficiency during the entire history of the black hole is

$$\bar{\varepsilon} \approx \frac{(\zeta - 1)\varepsilon_0}{\zeta - \varepsilon_0}.$$

Assume that during the sub-Eddington accretion phase the spin of the black hole is kept at the canonical value and hence the radiation efficiency is  $\varepsilon_0 \approx 0.3$ . Then, if  $\zeta > 1.35$ , we would have  $\bar{\varepsilon} > 0.1$ .

In this two-phase scenario for the cosmic growth of the black hole, the black hole gets weight through super-Eddington accretion, and does work through following-up sub-Eddington accretion. A sketch for a black hole to obtain a mass of  $10^9 M_\odot$  this way by redshift  $z = 6$  is presented in Fig. 2.

## 7 A SIMPLE MODEL FOR THE GROWTH OF BLACK HOLES IN QUASARS

In this section we consider a simple model for the growth of black holes in quasars. We assume that the mass accretion rate has the form  $\dot{M} \propto M_{\text{H}}^2 e^{-M_{\text{H}}/M_{\text{cr}}}$ , i.e., it is given by the Bondi rate when the mass of the black hole is small and starts to decay exponentially when the mass of the black hole approaches a critical value  $M_{\text{cr}} \sim 10^8 M_\odot$ . The decay in the mass accretion rate is assumed to be caused by the feedback effect (see Sec. 6). Then we have

$$\xi = \xi_0 m e^{-m/m_{\text{cr}}}, \quad (22)$$

where  $m_{\text{cr}} \equiv M_{\text{cr}}/M_{\text{H},0}$ .

We adopt the slim disk model for super-Eddington accretion, where the disk luminosity is  $\propto \ln \dot{M} \propto \ln m$  when  $\xi \gg 1$  (Abramowicz et al. 1988; Watarai et al. 2000; Watarai, Mizuno & Mineshige 2001). To make a smooth transition from sub-Eddington to super-Eddington accretion, we assume a simple formula for the disk luminosity

$$L = \alpha^{-1} \varepsilon_0 L_{\text{Edd}} \ln(1 + \alpha \xi), \quad (23)$$

where  $\alpha$  is a constant, and  $\varepsilon_0$  is the efficiency for a standard thin disk. When  $\alpha \xi \ll 1$ , equation (23) returns to the luminosity for a standard thin disk,  $L \approx \varepsilon_0 \xi L_{\text{Edd}} = \varepsilon_0 \dot{M} c^2$ . When  $\alpha \xi \gg 1$ , we have  $L \approx \alpha^{-1} \varepsilon_0 L_{\text{Edd}} \ln(\alpha \xi)$ . Comparing this with the results in Watarai et al. (2000) and Watarai, Mizuno & Mineshige (2001), we have  $\alpha \sim \varepsilon_0/2 \approx 0.15$  if we assume that  $\alpha_0 \approx 0.3$  corresponding to the maximum efficiency of a standard thin disk.

By equation (23), the efficiency of the disk is

$$\varepsilon = \frac{\varepsilon_0}{\hat{\xi}} \ln(1 + \hat{\xi}), \quad \hat{\xi} \equiv \alpha \xi. \quad (24)$$

Then, submitting  $E_{\text{in}}^\dagger = 1 - \varepsilon$  into equation (1), we get the equation that governs the evolution of the black hole mass

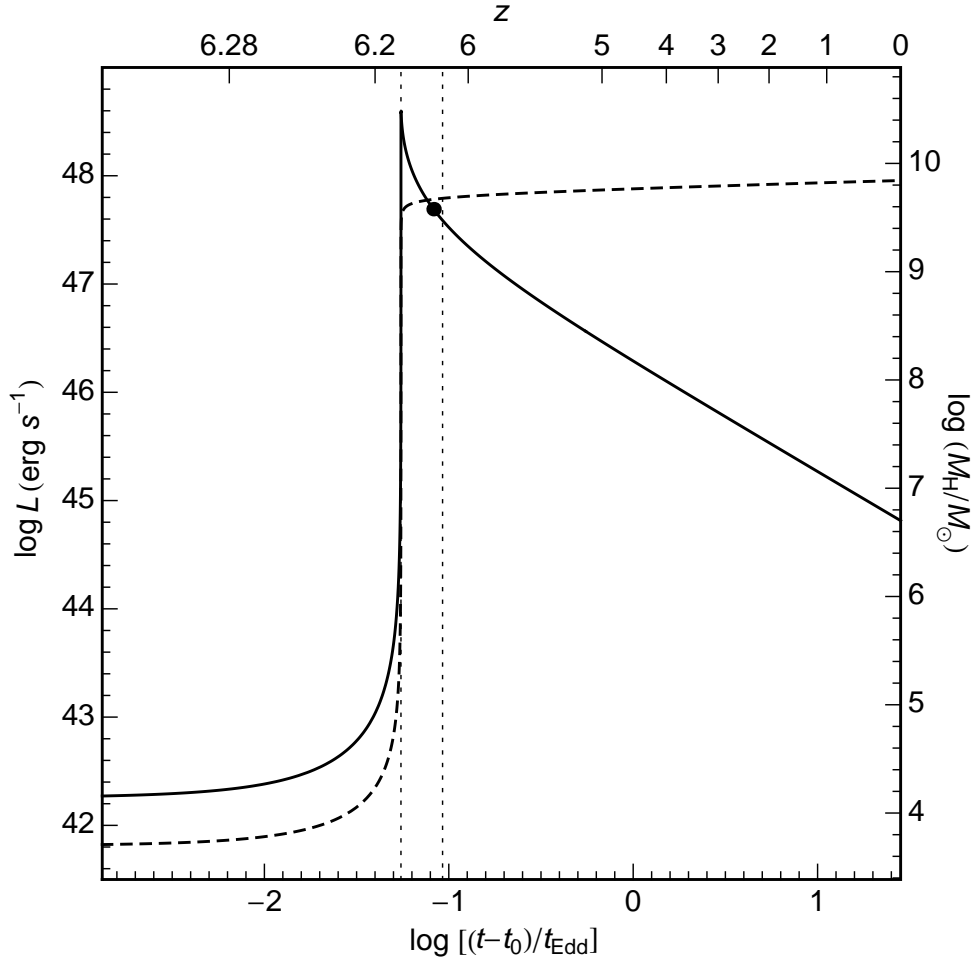
$$\frac{dm}{d\hat{\tau}} = m [\hat{\xi} - \varepsilon_0 \ln(1 + \hat{\xi})], \quad (25)$$

where  $\hat{\tau} \equiv \alpha^{-1} \tau$ .

We set the parameters as follows:  $\alpha = 0.15$ ,  $M_{\text{cr}} = 3 \times 10^8 M_\odot$ . At the cosmic time  $t_0 = 0.9$  Gyr, the initial mass of the black hole is  $M_{\text{H},0} = 5,000 M_\odot$ , and  $\xi_0 = 20$ . The black hole starts with a super-Eddington phase with  $\varepsilon_0 \xi = 6$  and a luminosity  $L \approx 2.8 L_{\text{Edd}}$ . Its mass evolves according to equation (25), supplemented by equation (22) and  $\hat{\xi} = \alpha \xi$ . As the black hole grows the accretion becomes more super-Eddington since  $\xi$  grows with time, until its mass becomes large enough ( $> M_{\text{cr}}$ ),  $\xi$  starts to decrease. When  $\xi$  becomes smaller than  $1/\varepsilon_0 \approx 3.3$ , the accretion enters the sub-Eddington phase during which the spin of the black hole is frozen at  $a_* = 0.998$ , and the accretion efficiency is fixed at  $\varepsilon_0 = 0.3$ .

The numerical integration of equation (25) is shown in Fig. 3, where the dashed curve shows the evolution of the black hole mass, the solid curve shows the evolution of the disk luminosity. The transition from super-Eddington to sub-Eddington takes place at  $t = 0.937$  Gyr (the dark point in the figure), where  $\varepsilon_0 \xi = 1$  and  $L \approx 0.81 L_{\text{Edd}}$ . During the super-Eddington phase (left to the dark point), the mass of the black hole grows by a factor of  $\sim 10^6$ . The luminosity of the disk peaks in the super-Eddington phase at





**Figure 3.** Evolution of the luminosity (solid line) and the mass (dashed line) of the black hole, for the toy model in Sec. 7. The parameters are:  $\alpha = 0.15$ ,  $M_{\text{H}}(t_0) = 5,000 M_{\odot}$ ,  $\xi(t_0) = 20$ , and  $M_{\text{cr}} = 3 \times 10^8 M_{\odot}$ . Super-Eddington accretion starts at  $t_0 = 0.9$  Gyr, and ends at  $t = 0.937$  Gyr (marked by the black dot). Then the accretion becomes sub-Eddington, with an efficiency  $\varepsilon_0 = 0.3$ . The luminosity peaks at  $t_{\text{peak}} = 0.925$  Gyr. The two vertical dotted lines bound the quasar epoch when the luminosity is above one-tenth of the peak luminosity. The time interval spanned by the two dashed lines is  $1.7 \times 10^7$  yr. On the top of the figure, the cosmic redshift is labeled for  $z = 0, 1, 2, \dots 6.28$  ( $t_0 = 0.9$  Gyr corresponds to  $z = 6.296$ ).

$t_{\text{peak}} \approx 0.925$  Gyr where  $\varepsilon_0 \xi \approx 1.4 \times 10^3$ , with a peak luminosity  $L_{\text{peak}} \approx 13 L_{\text{Edd}}$ . At the transition time the black hole has a mass of  $4.7 \times 10^9 M_{\odot}$ , and a luminosity of  $4.9 \times 10^{47}$  erg s $^{-1}$ .

During the sub-Eddington phase (right to the dark point), the mass of the black hole grows very slowly, and the disk luminosity decays quickly due to the drop in the mass accretion rate. By the time of today ( $z = 0$ ), the mass of the black hole would be  $7.0 \times 10^9 M_{\odot}$ , and the luminosity of the disk would be  $6.5 \times 10^{44}$  erg s $^{-1}$ .

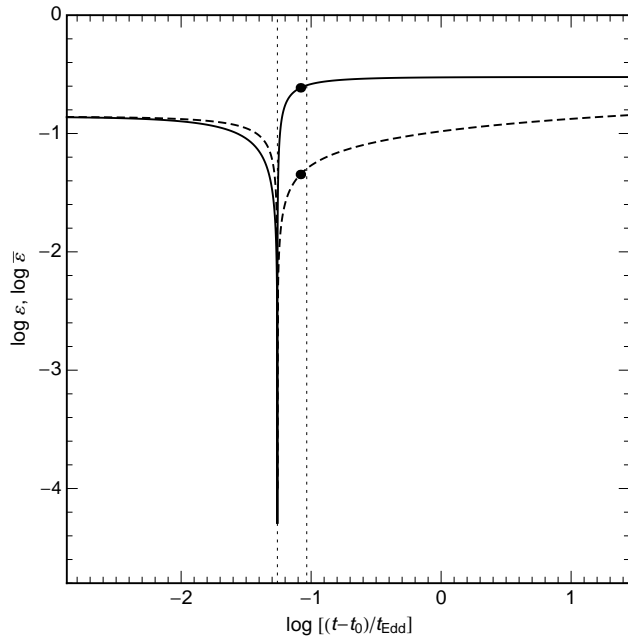
The time-duration in which the luminosity of the disk exceeds one-tenth of the peak luminosity (the region bounded by the two vertical dotted lines in Fig. 3) is  $\approx 1.7 \times 10^7$  yr. This is in agreement with the observational constraint on the lifetime of unobscured quasars:  $10^6 \text{ yr} \lesssim t_{\text{Q}} \lesssim 10^8 \text{ yr}$  (Martini 2004).

In Fig. 4 we show the instantaneous accretion efficiency (solid line) and the average accretion efficiency (dashed line), as a function of time. The average accretion efficiency is defined by equation (21).

The instantaneous efficiency  $\varepsilon$  minimizes at  $t \approx 0.92$  Gyr, when  $M_{\text{H}} \approx M_{\text{cr}}$ . When  $M_{\text{H}} > M_{\text{cr}}$ , the reduce in the mass accretion rate causes  $\xi$  to drop quickly, and the efficiency  $\varepsilon$  grows quickly. The average efficiency  $\bar{\varepsilon}$  minimizes at  $t \approx 0.94$  Gyr, when  $M_{\text{H}} \approx 8.1 \times 10^8 M_{\odot}$ . At the transition time, the instant efficiency becomes 0.24, and the average efficiency is 0.045. By the time of today, we have  $\varepsilon \approx 0.3$  and  $\bar{\varepsilon} \approx 0.14$ .

If in the definition of the average efficiency (eq. 21) the time integration is over only the duration when the luminosity exceeds one-tenth of the peak (i.e., over the region bounded by the two vertical dashed lines), then we obtain an average efficiency of 0.052.

The overall average efficiency  $\bar{\varepsilon} \approx 0.14$  is consistent with the observational constraints (Soltan 1982; Yu & Tremaine 2002; Elvis, Risaliti & Zamorani 2002; Wang et al. 2006).



**Figure 4.** Evolution of the instantaneous accretion efficiency (solid curve) and the average accretion efficiency (dashed curve) for the model in Fig. 3. The average efficiency is defined by the total energy emitted by the disk integrated from the start of the super-Eddington accretion (at  $t_0 = 0.9$  Gyr) to a later time, divided by the total mass accreted during that time interval (eq. 21). As in Fig. 3, the black dots mark the transition from super-Eddington to sub-Eddington accretion, and the two vertical dotted lines bound the quasar epoch.

## 8 CONCLUSIONS

We have proposed a simple model for the rapid growth of supermassive black holes in quasars. In this model, a major merger of two galaxies (or some secular processes like global disk instabilities) brings a large quantity of cold gas to the region around the central black hole formed by the coalescence of the two black holes in the nuclei of parent galaxies. Due to the very high density of the cold gas, a super-Eddington accretion phase is turned on. During this super-Eddington phase, the mass of the black hole inflates quickly, without the feedback effect all the mass of the gas would have been swallowed in a fraction of the Eddington time. The existence of black holes of masses  $\sim 10^9 M_\odot$  at  $z \sim 6$  can easily be explained with this model.

When the mass of the black hole becomes large enough ( $\gtrsim 10^8 M_\odot$ ), the feedback effect becomes important which shuts off the super-Eddington accretion. Afterwards, the black hole enters a sub-Eddington accretion phase, during which the mass of the black hole continues increasing but with a slower rate, until all of the surrounding gas are taken over or a next event of merger/instability happens.

Although the super-Eddington phase has a very low efficiency in converting mass into radiation, the subsequent sub-Eddington phase has a very high efficiency ( $\approx 0.3$  for a standard thin disk around a nearly maximal-spinning black hole). If during the later sub-Eddington phase the mass of the black hole gets boosted by a factor  $\gtrsim 1.5$  then the overall average efficiency will be  $\gtrsim 0.1$ , consistent with the observational constraint on the average accretion efficiency

of quasars (Yu & Tremaine 2002; Elvis, Risaliti & Zamorani 2002; Wang et al. 2006).

In our treatment of super-Eddington accretion we have ignored the effect of disk outflows. The judgment for our choice is based on the fact that to prevent the disk luminosity from exceeding the Eddington luminosity the mass accretion rate does not have to be reduced since the disk can adjust itself to have a low efficiency either through photon trapping in the inner region of the disk, and/or through pushing the inner boundary of the disk toward the marginally bound orbit. Of course this is an issue under debate and further detailed investigation is needed to determine if a strong outflow will be driven by a super-Eddington accretion so that the majority of the gas accreting onto the black hole will be expelled to infinity by the radiation pressure. However, a recent study by Wyithe & Loeb (2011) indicates that this will not happen at least in certain circumstances, where the photon diffusion is too slow to expel the accreting gas.

The scenario that we have proposed is consistent with the popular view that galaxy mergers, quasars, and the growth of supermassive black holes coevolve (Di Matteo, Springel & Hernquist 2005; Hopkins et al. 2005, 2006, 2007; Sijacki et al. 2007; Di Matteo et al. 2008; Treister et al. 2010; Debuhr, Quataert & Ma 2011).

It would be interesting to test the scenario with observations. Observational evidence for super-Eddington accretion in quasars has been discussed by Collin et al. (2002). Recently, Kawakatu & Ohsuga (2011) proposed a new method to explore super-Eddington accretion in AGNs by near-infrared observations. They found that generally the ratio of the AGN IR luminosity and the disk bolometric luminosity for super-Eddington AGNs is much smaller than that for sub-Eddington AGNs, caused by the self-occultation effect of the super-Eddington accretion flow. While for nearby galaxies currently undergoing major mergers, super-Eddington may be observed by direct or indirect measurements of the mass accretion rate and the luminosity like in the case of Narrow-Line Seyfert 1 galaxies (Kawaguchi 2003).

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